A finite element model updating approach intended to simulate vibro-acoustic behaviour of aluminium foam structures

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Aluminium foam is a promising material for reduction of structure borne sound due to its high ratio of stiffness to mass. Thus, from the acoustic point of view, the use of Al-foam in structures excited by dynamic forces, e.g. machine housings, is attractive. Based on this observation a procedure is developed using the finite element method in order to simulate structure borne sound of Al-foam.

The approach employs a continuum theory with the aim to model the structure's material properties. In contrast to FE-models commonly used, the approach enables one to take account of the inhomogeneous stiffness and density distributions throughout the foam:

A FE-model based on average material properties is created first. With the aim to increase accuracy of numerical predictions, eigenfrequency measurements are performed, using the foam structure to be modelled. The obtained test data are taken in order to improve (update) the original FE-model. Hence the approach is related to the area known as "model updating".

The updated FE-model, showing a discrete approximation of the physical stiffness and density distributions, can be used as a means of non-destructive testing by material property identification. Material property identification of this kind reveals potential applications within quality control and damage detection.

1 Introduction

In this paper Al-foams consisting of closed cells are considered. Within these foams no relative motion between air and skeleton is possible. Frictional losses resulting from this mechanism, typical for foams showing high sound absorption, do not occur. Hence, even though usually sound absorption is of main interest with regard to acoustic applications of foams, a different application will be considered here.

Time dependent dynamic motions - i.e. inertia forces are taken into account - in the acoustic frequency range are denoted as "structure borne sound". Structure borne sound on the surface of a structure leads to radiation of sound into the surrounding air. For noise control it is of prime importance to reduce structure borne sound. The potential of Al-foam to reduce structure borne sound is high, by the following reasoning.

With Young's modulus E, plate thickness h and Poisson's number ν the bending stiffness B of a plate structure equals [1]

$$B = \frac{E\,h^3}{12\,(1-\nu^2)} \tag{1}$$

Al-foam shows a relation between Young's modulus and foam density ρ which may be written by introducing a material dependent parameter κ

$$E \propto \rho^{\kappa} \tag{2}$$

J. Banhart, M.F. Ashby, N.A. Fleck: *Metal Foams and Porous Metal Structures.* © MIT Verlag (1999)

Bending stiffness is proportional to the plate thickness to the power of three while Young's modulus is proportional to the density to the power of κ. The parameter κ is less than 2 for the foams considered here. Consequently, with the aim of increasing bending stiffness, the wall thickness of foam structures can be enlarged whereas the foam density has to be reduced simultaneously in order to retain the structure's mass. Of course, within practical applications there are certain limits with regard to this procedure, i.e. geometrical restriction to the maximum wall thickness and technical restrictions concerning the minimum foam density.

Concluding, it can be stated that replacing materials commonly used in mechanical engineering by Al-foam, structures showing high bending stiffness can be obtained. An increased ratio of bending stiffness to weight leads to a high impedance, i.e. the structure's resistance against vibrations excited by forces, is high. Furthermore, the quasistatic frequency range, i.e. the range up to the first eigenfrequency, increases. With regard to sound radiation both effects are strongly desired.

The objective of this paper is to present an attempt suited to model structure borne sound of Al-foam in order to exploit the material's potential efficiently. For modelling structure borne sound of Al-foam the finite element method will be used. The procedure proposed here can be characterised by two steps:

1) A homogenisation approach is employed in order to enable numerical calculations concerning vibrations of Al-foam structures. The approach is suited to assess the dynamic behaviour of Al-foam structures as well as to optimise such structures with respect to acoustic properties. The procedure does not take into account inhomogeneous material property distributions.
2) Based on the homogenisation approach an updating approach is developed. This approach enables modelling of Al-foam in a different, refined way: Within a first step measurements have to be performed using the foam structure which shall be modelled. The test data are used to improve the original FE-model with the aim to increase the model's accuracy. Hence the updating approach can only be applied on condition that the real foam structure is already available.

2 Homogenisation Approach

The homogenisation approach enables macroscopic modelling of Al-foam using the theory of continuum mechanics. The approach is based on the following assumptions:

- The foam shows a random topology leading to macroscopically isotropic behaviour.
- Linear acoustics is considered, i.e. the occurring strains can be characterised as small. A linear-elastic material law can be used.
- The cells are closed, i.e. no relative motion between gas and skeleton can occur.
- The damping forces are low and can be modelled by viscous damping.
- The foam's elastic behaviour is dominated by the skeleton's properties while the contribution of the gas within the pores can be neglected.

On these conditions a homogenisation which does not take account of coupling effects between solid and fluid can be introduced. Consequently, the foam structure can be modelled as an linear-elastic, isotropic continuum. It can be described using average values for Young's modulus, Poisson's number, density and loss factor. All these values can be determined experimentally.

Applying the homogenisation approach numerical calculations can be performed employing the finite element method. From the acoustic point of view bending waves are of primary importance due to their dominating influence on acoustic radiation. In order to enable accurate modelling of bending-dominated problems a second order continuum element is chosen, which is known to be very effective for this kind of problems.

Due to the low material damping of Al-foam (η < 0.01) modal properties, i.e. eigenfrequencies and mode shapes, can be computed easily with the knowledge of the structure's mass- and stiffness-matrices only, by solving the eigenvalue problem of structural dynamics. Comparing calculated modal properties with measured ones reveals some deviations obviously resulting from structural inhomogeneities (see also results). In order to account for inhomogeneity within the FE-model an updating approach is developed.

3 Updating Approach

In traditional analysis of structural systems it is usually assumed that the material properties are constant. However, especially as modern engineering structures are becoming increasingly complex by employing advanced materials, it is evident that models have to take into account existing inhomogeneity. With regard to structures produced from of Al-foam, the dynamic behaviour can be modelled with improved accuracy taking into account an inhomogeneous stiffness and density distribution throughout the structure.

For this reason the homogenisation approach used to simulate structure borne sound fields is extended. An updating formulation is developed. In general the area known as model updating [2-6] is concerned with the correction of finite element models by using measured test data (see also Fig. 1).

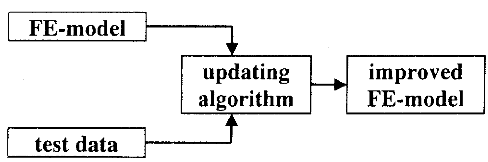

Fig. 1: Updating of FE-model

For the purpose of updating the model obtained from applying the homogenisation approach, appropriate test data have to be selected. Since the structure's eigenfrequencies can be easily and accurately determined by measuring at only one point of the structure, a method based on the measurement of eigenfrequencies is potentially very attractive in comparison to other techniques. To get reliable results only the lower eigenfrequencies can be used which can be easily assigned to their corresponding eigenmodes. Hence only a small number of eigenfrequencies can be measured directly. For the purpose of enlarging the number of measurable eigenfrequencies, a so-called multi boundary condition test (MBCT) is introduced [3]. The basic idea of the MBCT is to use different boundary conditions which lead to different eigenfrequencies and corresponding mode shapes. Thus, the information obtained about the structure can be increased. With regard to Al-foam structures a MBCT can be realised by clamping the structure in different positions. An example is shown in Fig. 2.

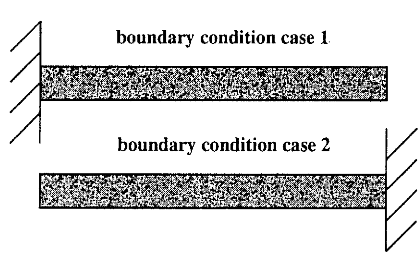

Fig. 2: Clamping of Al-foam beam using different boundary condition cases

Assuming the eigenfrequencies to be measured exactly the updating approach is done with the intention to minimise a distance function between the calculated eigenfrequencies and the corresponding measured eigenfrequencies. In order to reach this objective appropriate material parameters have to be taken into account within the updating procedure. In principle the material properties used for continuum modelling all depend on location **x** due to the material inhomogeneity.

$$E = E(\mathbf{x}) \quad ; \quad \nu = \nu(\mathbf{x}) \quad ; \quad \rho = \rho(\mathbf{x}) \tag{3}$$

Consequently the system's output data, the eigenfrequencies, are known, while the material parameters of the model itself have to be identified. Problems of this kind can be classified as inverse problems. Inverse problems can be satisfied by more than one solution in principle. For the purpose to obtain a unique solution the following restrictions are introduced:

The Poisson ratio of a foam depends only on cell topology but not on foam density [7]. The foams investigated do show no kind of regular cell topology. Since the material consists of cells with much smaller dimensions than the structure itself a lot of different cells are involved, thus Poisson ratio will result as a function of an average cell geometry. Hence the Poisson ratio is assumed to be constant.

Young's modulus and density can not be identified by modal test data simultaneously. Therefore a relation must be introduced allowing to couple these parameters. The foam's dependency of Young's modulus can be approximated by

$$E_{\text{foam}} = E_{\text{solid}} \left(\frac{\rho_{\text{foam}}}{\rho_{\text{solid}}} \right)^{\kappa} \tag{4}$$

where ρ_{solid} and E_{solid} denote density and Young's modulus of aluminium. Thus identifying density in dependency of location Young's modulus is determined also.

Assuming a continuous density- and stiffness distribution, $\rho(\mathbf{x})$ can be approximated by introducing an interpolation scheme. The interpolation is performed similarly to those of the FEM. However, for this interpolation a coarser grid is used than for the FEM computation. Related to the grid's nodes discrete density parameters ρ_A with corresponding shape functions $N_A(\mathbf{x})$ are defined. Multiplying the discrete density values with their corresponding shape functions and summarising the products a function $\rho^h(\mathbf{x})$ can be obtained describing density in dependence of location:

$$\rho^h(\mathbf{x}) = \sum_A N_A(\mathbf{x})\, \rho_A \tag{5}$$

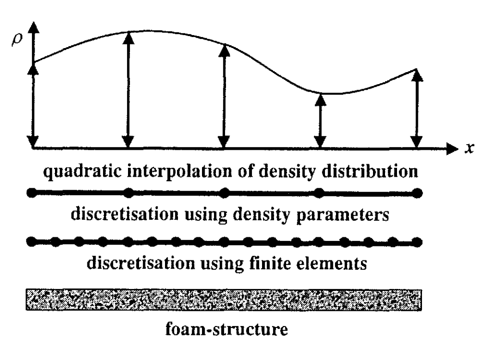

Fig. 3: Example of interpolation using discrete density parameters

With the intention to assign a different value of density to each finite element the value of $\rho^h(\mathbf{x}^e)$ at the centeroidal location $\mathbf{x}^e$ of the respective element can be used. In Fig. 3 a beam structure is depicted in order to illustrate the interpolation scheme. The beam is modelled using 16 finite elements while 5 discrete density parameters have to be determined. Between these parameters a quadratic interpolation is used in order to obtain the density distribution of the whole structure.

4 Iterative Updating-Algorithm

To solve the problem for the unknown density-parameters ρ_A an iterative numerical algorithm is formulated. Denoting the iteration index with μ the basic steps are listed below.

1) Following the homogenisation approach a homogeneous density distribution is assumed to begin with. Young's modulus is calculated according to Eq. (4) using the average foam density ρ_{ave}

$$\rho^{[\mu=1]} = \rho_{ave} \quad ; \quad E^{[\mu=1]} = E_{solid}\left(\frac{\rho_{ave}}{\rho_{solid}}\right)^{\kappa} \tag{6}$$

2) Mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ are computed and the corresponding eigenvalue problem is solved for the eigenfrequencies $f_i^{[\mu]}$ and mode shapes $\phi_i^{[\mu]}$.

$$(\mathbf{K}^{[\mu]} - 4\pi^2 f_i^{[\mu]2}\,\mathbf{M}^{[\mu]})\,\mathbf{\Phi}_i^{[\mu]} = 0 \tag{7}$$

3) The vector of calculated eigenfrequencies $\mathbf{f}$ is subtracted from the corresponding measured one $\mathbf{f}^T$

$$\Delta\mathbf{f}^{[\mu]} = \mathbf{f}^T - \mathbf{f}^{[\mu]} \tag{8}$$

4) The derivatives of the eigenfrequencies with respect to the density parameters are computed in order to assemble the sensitivity matrix. Regarding m eigenfrequencies and n density parameter a $m\times n$ sensitivity matrix results. Due to the use of the MBCT-concept the eigenfrequencies are related to different boundary conditions. Thus, the sensitivity matrix $\mathbf{S}$ has to be assembled out of $o\times n$ submatrices $\mathbf{S}^k$ where k denotes the respective boundary condition case and o the number of related eigenfrequencies.

$$(\mathbf{S}^k)^{[\mu]} = \left.\frac{\partial f_i^k}{\partial \rho_j}\right|_{\rho^{[\mu]}} \tag{9}$$

5) With the aim to obtain an accurate parameter correction vector $\Delta\rho$ an overdetermined system of linear equations is set up, i.e. the number of eigenfrequencies taken into account is greater than the number of unknown parameters

$$(\mathbf{S}^k)^{[\mu]}\,\Delta\boldsymbol{\rho}^{[\mu]} = (\Delta\mathbf{f}^k)^{[\mu]} \tag{10}$$

The overdetermined problem is solved for the discrete density parameters in the least squares sense. Due to the change of density parameters the model's mass m is changed also. In order to minimise the difference between the model's mass and the measured mass of the structure m^T, a further equation is included in the least squares problem. Thus, the problem is

$$\min_{\Delta\rho_j^{[\mu]}} \left\| \begin{matrix} \Delta\mathbf{f}^{[\mu]} - \mathbf{S}^{[\mu]}\Delta\boldsymbol{\rho}^{[\mu]} \\ m^T - m^{[\mu]}\,(\Delta\boldsymbol{\rho}^{[\mu]}) \end{matrix} \right\|_2 \tag{11}$$

where $||...||_2$ denotes the Euclidean or L_2-norm. With the correction vector $\Delta\rho^{[\mu]}$ the density parameters of the following iteration can be determined

$$\boldsymbol{\rho}^{[\mu+1]} = \boldsymbol{\rho}^{[\mu]} + \Delta\boldsymbol{\rho}^{[\mu]} \qquad (12)$$

6) Using the density parameters the density of the respective finite element ρ^e can be interpolated. The related Young's modulus has to be calculated according to Eq. (4)

$$\rho^{e[\mu+1]} = \sum_j N_j(\mathbf{x}^e)\, \rho_j^{[\mu]} \qquad (13)$$

7) If the iteration's convergence criteria are fulfilled the algorithm stops, otherwise the next iteration starts with step 2).

5 Results

For the purpose of validating the above described technique for identifying density and stiffness distribution, a beam structure made of foam produced by Hydro Aluminium was chosen. The structure with the dimensions 480mmx35mmx35mm shows an average density of 281 kg/m^3. Clamping the structure in 2 different positions each time the eigenfrequencies corresponding to the lower 4 bending-modes were measured, i.e. 8 eigenfrequencies were used with the aim to update the FE-model.

The measurements were performed using an impact hammer in order to excite the structure with a pulse force. The force was measured by a force transducer. The velocity of the beam's free flexural vibrations was monitored by a laser vibrometer. Using a Fast Fourier Transform algorithm the transient velocity and force signals were transformed to the frequency domain and taken to calculate the mobility function $M(\omega)$.

$$M(\omega) = \frac{v(\omega)}{f(\omega)} \qquad (14)$$

where $v(\omega)$ and $f(\omega)$ are the Fourier transforms of the velocity and force, respectively.

Due to a high resolution of 0.8 Hz within the frequency domain an accurate determination of eigenfrequencies was possible by identifying peaks of the mobility function. The test set-up is depicted in Fig. 4. The first clamping position is shown; the second clamping position was chosen symmetrically to the first one. In both cases the measurements concerning the upper and the lower part of the clamped structure were done separately.

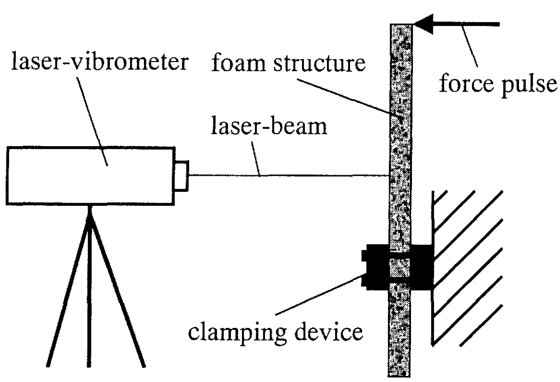

Fig. 4: Test set-up

The beam structure was discretised by 32 brick elements with 27-nodes. The discretisation using density parameters was done according to Fig. 3, i.e. 2 elements with quadratic shape functions (5 parameters) were chosen. Employing the updating algorithm the average difference of measured and calculated eigenfrequencies could be decreased from 2.8% within the first iteration to 0.7% within the fifth iteration. Thereby the algorithm identified a suiting density- respectively stiffness distribution. The measured eigenfrequencies, those of the original FE-model and those of the updated one are shown in Table 1. They are listed in dependency of their related boundary condition case. The results reveal that it was possible to reduce the differences of nearly all eigenfrequencies. Especially the eigenfrequency of the first bending mode was changed strongly in order to fit to the measured one.

boundary condition case	measured eigen-freq. [Hz]	original model (1. Iter.)		updated model (5. Iter.)	
		calculated eigenfreq. [Hz]	error [%]	calculated eigenfreq. [Hz]	error [%]
1	318	283.0	11.0	317.3	0.2
	1070	1092	2.1	1070	0.0
	1647	1643	0.2	1663	1.0
	4098	4166	1.7	4095	0.1
2	276	283.0	2.5	274.2	0.7
	1091	1092	0.1	1090	0.1
	1619	1643	1.5	1656	2.3
	4307	4166	3.3	4254	1.2

Table 1: Comparison of measured and calculated eigenfrequencies

For the purpose of verifying the calculated density distribution the beam structure was cut into 10 pieces of equal length. A measured density distribution related to the piece's volume was thus obtained. Regarding the structure's measured and calculated density distribution shown in Fig. 5 reveals a high degree of correlation. Especially the reduced density (and stiffness) at the end of the beam could be identified clearly.

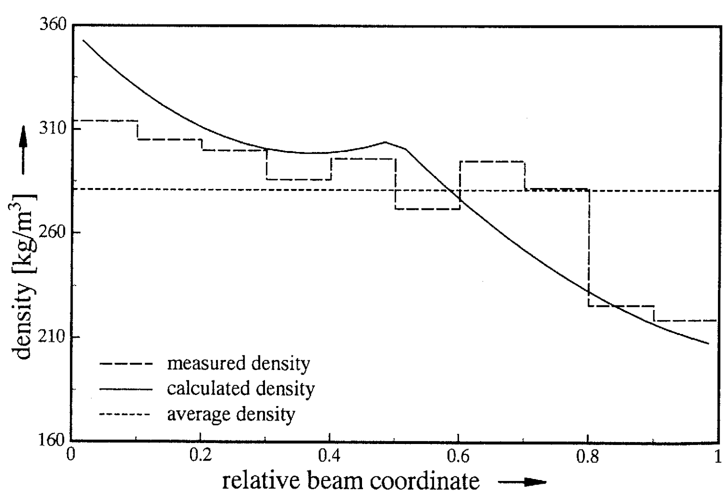

Fig. 5: Comparison between measured and calculated density distribution

The reduced foam density at the end of the beam also enables a physical explanation concerning the strong deviation of eigenfrequencies considering the first bending mode before employing the updating algorithm. This mode shows a shape leading to a maximum acceleration at the end of the beam. Consequently the inertia forces within this region are very high. They give rise to a high sensitivity of the first mode's eigenfrequency related to the local density within this region, i.e. changing the foam density at the end of the beam entails a strong change of the first eigenfrequency.

6 Conclusions

Employing the finite element method this paper demonstrates how concepts from continuum theory can be used to model vibro-acoustical behaviour of Al-foam. Moreover with the aim to take account of the foam's inhomogeneity within the model an updating procedure based on an iterative solution of a non-linear least squares problem is developed.

Test results show that the proposed procedure enables to increase accuracy of dynamic calculations as well as to detect, locate and roughly quantify density- and stiffness changes within the foam structure. Thus, the method also reveals potential concerning non-destructively testing of Al-foam structures. Nevertheless in order to assess the method's capability of non-destructively testing further tests will have to be performed. Even though the updating approach allows to identify structural properties in more than one dimension this has to be verified especially.

Finally considering the procedure's performance within practical applications the following items have to be pointed out:

- The reliability of the relation describing the dependence of Young's modulus on density is of prime importance.
- The number of eigenfrequencies which can be identified and assigned to a certain mode shape directly affect the model's accuracy.
- Only a high accuracy of eigenfrequency measurements assures the identification of structural properties with physical meaning.

Acknowledgments

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